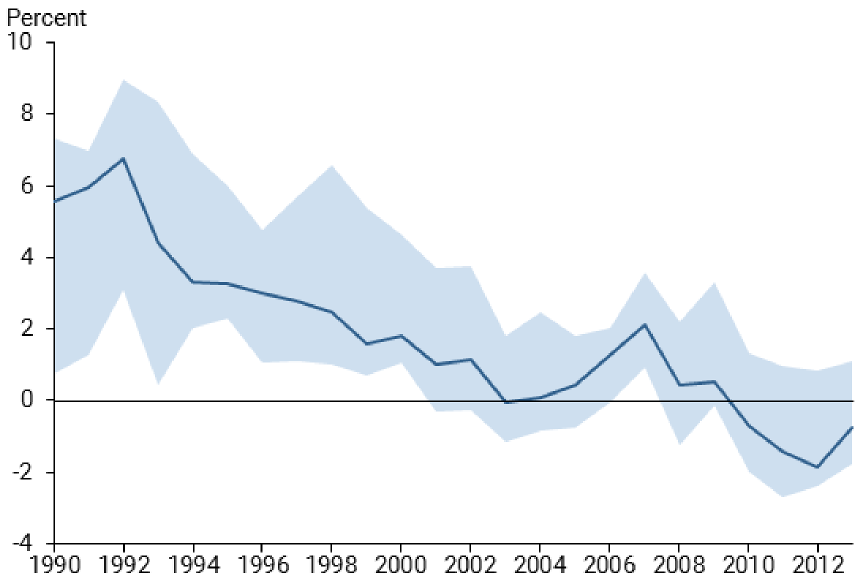
**Real Interest Rate in Developed Economies**

**Median and Range**




Source: Federal Reserve Bank of San Francisco

See the note at the end of article.

# A Lower Bound on Real Interest Rates

## By Jesse Aaron Zinn

Peer Reviewed

Jesse Aaron Zinn is an Assistant Professor of Economics in the College of Business at Clayton State University. His research interests include macroeconomics, public finance, and developmental economics. He earned his PhD in Economics at University of California, Santa Barbara.


## Abstract

In this short article, the author demonstrates that real interest rates cannot be lower than -1 (i.e. -100%). He discusses how the textbook approximation to the Fisher equation can lead to the erroneous belief that there is no lower bound on real interest rates. He also speculates that the widespread use of the Fisher equation is why we had yet to realize that a lower bound on real rates exists. This leads him to advocate using and teaching the exact Fisher equation, rather than its approximation.


## Introduction

Negative real interest rates have been common in the United States and other countries since the financial crisis of 2007-8. In fact, Japan has often had negative real interest rates going back to the early 1990s. In some countries, such as Denmark and Switzerland, there have been cases where *nominal* interest rates have also turned negative, disproving the idea that zero is an impenetrable lower bound on nominal rates. These phenomena have, no doubt, lead many to wonder how low rates can go.

The objective of this short piece is to demonstrate the existence of a lower bound on real rates of interest. Specifically, real rates cannot be less than -1 (that is, -100%). Seeing as negative real rates are a characteristic of both liquidity traps (Krugman, 1998; Eggertsson and Krugman, 2012) and secular stagnation (Summers, 2014, 2015), a lower bound on real interest rates suggests there is a limit to the severity of at least one dimension of these maladies.

The -1 lower bound on real rates of return manifests when an investment cannot result in a loss greater than its principal.[1] Theoretically, there are two ways for an investment to be worthless. The first way is for it to pay off zero and have no salvage value, in which case the nominal rate of return is -1. The second way for an investment to be worthless is if whatever it pays out (including salvage) loses all value, which is the case if a bond pays out a currency that has experienced infinite inflation and, therefore, lost all worth. Either way, the real rate of return approaches -1, as I show below.

It is surprising that before this work economists had yet to deduce a lower bound on real interest rates. Perhaps we did not notice it because we teach and sometimes use the approximation to the Fisher equation and this approximation has no lower bound.[2] Even high-ranking officials within the Federal Reserve have made the mistake of stating that there is no lower bound on real interest rates, and I suspect that they do so with the approximation to the Fisher equation in mind.[3] Because of all this, I advocate teaching and utilizing the actual Fisher equation, rather than its approximation. Taking such advice would preclude these types of oversights while helping to ensure that our students calculate real rates of return with a higher degree of accuracy.

## Derivation of a Lower Bound on Real Interest Rates

Consider any asset with a nominal rate of return of $i \geq -1$. This restriction limits assets to those where the investment cannot result in a loss greater than its principal. Since the nominal rate of return of an investment that pays back zero and has zero salvage value is -1, we are simply assuming that the lowest possible nominal rate of return occurs when an investment results in no yield and a total, unsalvageable loss. This assumption holds for investments considered in the vast majority of macroeconomic studies. Examples of such investments include spending on land, physical capital, inventories, and R&D, as well as holding currency, lending money, and buying bonds. Moreover, in light of the fact that many economists see zero as the lower bound for the nominal rate on bonds, the assumption that $i \geq -1$ should not raise much controversy when focusing exclusively on bond rates.

Let $\pi$ represent the rate of inflation over the period it takes for the asset to pay off.[4] The Fisher equation determines the real rate of interest $r$ on the asset:[5]

$$1 + r = \frac{1 + i}{1 + \pi}.$$

Solving for the real interest rate yields

$$r = \frac{i - \pi}{1 + \pi}.$$

Using the Fisher equation we find that -1 is a lower bound for $r$ follows from the assumption that $i \geq -1$. To see this, subtract $\pi$ from both sides of $i \geq -1$ and divide the resulting differences by $1 + \pi$ to find that

$$\frac{i - \pi}{1 + \pi} \geq \frac{-1 - \pi}{1 + \pi},$$

which is equivalent to $r \geq -1$. To summarize this logic, we have found that

$$i \geq -1 \quad \text{implies} \quad r \geq -1,$$

for any value of $\pi$.[6]

## Two Ways to Approach the Lower Bound on *r*

The real interest rate $r$ can approach its lower bound of -1 in two ways. First, if price levels are positive then $\pi > -1$, which implies through the Fisher equation that $r$ is strictly increasing in $i$. Therefore, if $i$ reaches its minimum value of -1 then $r$ also reaches a unique minimum. This minimum for $r$ is -1 as well, which can be verified by substituting -1 for $i$ in the Fisher equation.

The second way $r$ can approach the lower bound of -1 is through increases in the rate of inflation. During episodes of hyperinflation (such as Germany in 1923, Hungary in 1946, and Zimbabwe in 2008) rates of inflation have been astronomical, being dozens of orders of magnitude above what economists consider normal or healthy. The scales of these hyperinflations suggest that for any value of $\pi$ there is no reason $\pi$ cannot be one percentage point higher. Moreover, we can think of a currency's collapse (where it becomes worthless) as a case of infinite inflation. Therefore, it seems reasonable to consider the implications of $\pi$ increasing without bound. Taking the limit of $r$ through the Fisher equation as $\pi$ goes to infinity, we find that $r$ goes to -1 for any finite $i$.

Note also that these two ways for $r$ to reach -1 are logically consistent in the sense that one does not preclude the other. Hence, it is possible for the two to work in tandem and bring $r$ to this lower bound more quickly than either effect on its own. This would be the case for an investment project that is liquidated for much less than what it initially costs (so, $i$ near -1) during a period of hyperinflation (when $\pi$ is very large).

## Remarks on Pedagogy

In practice, many economists think of real interest rates using the approximation to the Fisher equation $r \approx i - \pi$. Indeed, many introductory textbooks teach students that $r = i - \pi$, without the slightest warning that it is merely a rough calculation whenever inflation is non-zero.[7] If one takes $r \approx i - \pi$ as exact then it is straightforward to deduce that $r$ has no lower bound, since $i - \pi$ goes to negative-infinity as $\pi$ goes to infinity (for any finite $i$).

Taking the nominal interest rate as given, the approximation of the Fisher equation is always $1 + \pi$ times the exact calculation. This is why the approximation is accurate when $\pi = 0$ and why it becomes worse with greater magnitudes of inflation (or deflation). Thus, when inflation is 2%, so $\pi = 0.02$, the approximation is 1.02 times the actual real interest rate. This is negligible in many contexts, though I would not want my accountant or CFO to be consistently off by a factor of 1.02. However, with 100% inflation, so that $\pi = 1$, the approximation is twice the exact calculation – a substantial difference in most any context. Obviously, with hyperinflation the approximation is even more inaccurate.

I have shown that the Fisher equation implies a lower bound of -1 on real interest rates and this lower bound is approached either as the nominal rate decreases to -1 or as the rate of inflation increases without bound. That we have missed this lower bound, despite it being so simple to derive, suggests that perhaps the approximation to the Fisher equation has blinded us. Maybe we would have known about this lower bound sooner if we had not so widely used and taught the approximation to the Fisher equation. In using the approximation, we thought we were only giving up a bit accuracy for computational convenience, when in fact we may have also sacrificed understanding.

In order to minimize the likelihood of additional oversights and to ensure that our students make accurate calculations of real interest rates, I suggest that we stop using and teaching the approximation to the Fisher equation. Yes, the correct formula is marginally more complicated than the approximation to the Fisher equation, but it would not be any more difficult than many other calculations we expect students to master in introductory economics courses. If we can teach principles-level students to calculate the deadweight loss due to import tariffs or the level of real GDP in a Keynesian cross model, surely they can find

$$r = \frac{i - \pi}{1 + \pi}.$$

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**Note:** Real short-term rates in the above graph are calculated as yields on short-term government securities with maturity less than one year minus realized CPI inflation. Shaded area shows the 90-10% range across countries. The sample includes Belgium, Germany, Finland, France, Greece, Iceland, Ireland, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom, and the United States.

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[1] For this to be true, what was put into an investment must include any debt used in its financing. There is no lower bound on an individual’s *personal* losses on short sells or with leverage getting arbitrarily close to 100%.

[2] As noted above, the interest rates used in the calculation of the real rates in the graph at the top of the first page of this article were calculated using the approximation to the Fisher equation.

[3] The existence of a lower bound on real interest rates contradicts recent comments (Fischer 2016) at the 40th Annual Central Banking Seminar, sponsored by the Federal Reserve Bank of New York, where it was said, “nothing dictates that the natural rate of interest should be positive; *indeed*, *the natural rate has no effective lower bound*.” (I added the italics to emphasize the exact passage that is in contention. Note that the natural rate is the real rate of interest for which output is at potential and inflation is constant. Holston, Laubach, and Williams (2017) provide recent estimates of the natural rate for United States, Canada, the Euro Area, and the United Kingdom, finding declines in all four economies over the last few decades.) Given the context, it is clear that this remark from Fischer (2016) was an afterthought, unnecessary to the main point that the natural rate is not necessarily positive. Leading economists have, for the better part of a century, found it necessary to point out that the natural rate can be non-positive. See, for example, Keynes (1936) and Samuelson (1958).

[4] All results in this work hold for cases in which there is uncertainty about inflation, so $\pi$ could represent expected inflation.

[5] By definition, the real per-unit value of an investment is equal to $1 + r$. If $\pi$ is the inflation rate over the term of the investment then the price level when the investment pays off is $1 + \pi$ times the price level at the time the investment was made. Therefore,

the real value of a one-unit investment that pays a nominal rate $i$ is $(1 + i)/(1+ \pi)$. Setting these equal to each other yields the Fisher equation.

[6] The converse is also true: $r \geq -1$ implies $i \geq -1$; if one knows that the real rate of return on some asset is no less than -1 then the nominal rate must also be no less than -1.

[7] See, for example, the definitions of "real interest rate" in Mankiw (2014) and Krugman and Wells (2015).